FE Exam – Fluids Review October 23, 2013

Important Concepts

Density, specific volume, specific weight, specific gravity (Water 1000 kg/m^3, Air 1.2 kg/m^3)
Meaning & Symbols?

Stress, Pressure, Viscosity; Meaning & Symbols?

- Normal Stress = -Pressure, Shear Stress = Viscosity * Shear Rate
- Viscosity (Absolute Dynamic Viscosity), Kinematic Viscosity = Viscosity/Density (Symbol?)

Pressure in a Static Fluid (Absolute, Gage), Head; Meaning & Symbols?

- Increases linearly with depth – slope equal to fluid specific weight
- Applications
  - Force on Submerged Surface:
  - Buoyancy Force:
  - Manometer:

Flows:

- Velocity:
- Steady:
- Incompressible:
- Inviscid:
- Laminar:
- Turbulent:
- Volumetric Flow Rate:
- Mass Flow Rate:
- Bernoulli Equation (Energy):
- Reynold’s Number (Dimensional Analysis):
- Impulse-Momentum Principle (Forces):

Applications:

Laminar flow – velocity distribution:

Drag, Drag Coefficient:

Pipe Losses (Moody Diagram):

Pump Power:

Pitot Tube:

Fluid Forces on Pipes, Bends, Enlargements, Contractions, Deflectors, Blades:

Other Issues:

Mach Number:

Orifices:

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Application: A Comprehensive Problem

A pump is used to generate a flow of water at 25^{o}C from the reservoir through a pipe and a horizontal nozzle and into a deflector located above and to the right of the reservoir. The deflector redirects the horizontal flow from the nozzle into a vertical flow. The redirection of the flow is done in such a way that the cross-sectional area of the vertical flow is the same as that of the flow area leaving the nozzle. The flow from the nozzle exerts a horizontal force, F, of 1 kN on the deflector. The nozzle is smooth with an outlet diameter, d, of 3 cm, and an inlet diameter, D, of 10 cm. The pipe also has a diameter, D, of 10 cm, and has a roughness factor of 0.06 mm. The pipe length, L, is 40 m. Any bends in the pipe are negligible. The nozzle height above the pipe entrance to the reservoir, H, is 10 m. The pipe entrance protrudes into the reservoir. The pipe entrance is a height, h, of 1 m above the bottom of the reservoir. The pump is positioned near the pipe entrance. The difference in pressure across the pump is 800 kPa. The efficiency of the pump, η, is 0.80. The left side of the reservoir includes a vertical gate. The gate is pivoted at its bottom, at a point level with the bottom of the reservoir. The width, w, of the gate is 4 m. The height of the gate extends above the upper surface of the water in the reservoir. A strut placed on the outside of the gate prevents the gate from opening. The strut makes an angle, θ, of 30^{o} with the horizontal. The vertical distance, Y, of the strut attachment point above the gate pivot is 3 m. The reservoir is large enough that the height of water in the reservoir can be taken to be constant. The water can be taken to be incompressible with constant density. The local acceleration of gravity is 9.8 m/s^{2}. Using the Fundamentals of Engineering Supplied-Reference Handbook as your sole resource, attempt to:

1. Use the following page to draw a sketch of the system, labeling all relevant system characteristics.
2. Determine the velocity of the flow leaving the nozzle.
3. Determine the volumetric flow rate through the nozzle.
4. Determine the velocity of the flow entering the nozzle.
5. Determine the gage pressure at the entrance to the nozzle.
6. Determine the pump power.
7. Determine the required gage pressure at the pipe entrance. Include frictional and minor losses.
8. Determine whether the flow in the pipe is turbulent or not.
9. Determine the height above the pipe entrance of the reservoir water level.
10. Determine the force in the strut required to keep the gate in its vertical position.

System Figure:

Method of Solution:

Work from what is known toward what you would like to know. Use the remainder of this page to outline your approach to solving this problem. Upon completion of your outline, proceed to the following page to review the recommended solution process. The solution process sequentially identifies the critical concepts, principles, and results. In reviewing the solution process, do your best to add the appropriate figures in the spaces provided. Good luck!

Concepts	Principle	Figure	Results
Force	Law of Action and Reaction. Horizontal force exerted by water on deflector is equal in magnitude but opposite in direction to the horizontal force exerted by the deflector on the water. From the force on the deflector we can determine the force on the water. From the force on the water we should be able to learn something about the flow of the water.		Horizontal force on water from the deflector is 1 kN toward the left (negative X-direction).
Control Volume, Entrance and exit areas.	We will consider the region occupied by water from the nozzle outlet through that position on the deflector where the flow has become completely vertical. We have water entering the control volume with some average horizontal velocity over an area equal to the area of the nozzle outlet. We have water leaving the control volume with some average vertical velocity but occupying that same area (given characteristic of the deflector). The region where water enters the control volume (nozzle outlet) will be denoted O for outlet. The region where water leaves the control volume will be denoted W for wall.		Water entrance and exit areas for this control volume are given to be equal and are: $A_O = A_W = \frac{\pi}{4} \cdot d^2 = 7.07 \cdot cm^2$
Incompressible, Volumetric flow rate, Velocity	Conservation of mass – continuity equation. For a flow with constant density (pages 39-40), the volumetric flow rate is constant and is equal to the product of the flow area and the average flow speed. As the entrance and exit areas are equal to one another, the entrance and exit flow speeds are also equal to one another. A horizontal flow with speed V enters the control volume through the given area while a vertical flow with that same speed exits the control volume through an area of the same magnitude.		The volumetric flow rate entering the volume is related to the entrance area and the average flow speed by: $Q = V \cdot A_O$ The entering flow is in the horizontal direction, while the exit flow is vertical.

Force, Momentum, Density	Impulse-Momentum Principle. Net force on volume is equal to the net rate at which momentum is leaving the volume(pages 41-42). We are interested only in the X-component of the equation. We can use the known force on the fluid, the known density (page 44, 997 kg/m^3), the above relationship between volumetric flow rate, flow area, and average flow velocity, along with the known area to determine the flow velocity at the nozzle exit and the volumetric flow rate.		$\sum \vec{F} = Q_2 \cdot \rho_2 \cdot \vec{V}_2 - Q_1 \cdot \rho_1 \cdot \vec{V}_1$ $-F = -A_O \cdot \rho \cdot V^2$ $V = \sqrt{\frac{F}{\rho \cdot A_O}} = 37.7 \cdot \frac{m}{s}$ $Q = A_O \cdot V = 0.0266 \cdot \frac{m^3}{s}$ $Q = 422 \cdot \frac{gal}{\min}$
Control Volume, Incompressible, Volumetric flow rate, Velocity, Area	Now that we know the conditions at the nozzle exit, it makes sense to consider a control volume bounded by the nozzle. We have horizontal flow entering the volume through the larger nozzle inlet area (denoted I) and exiting through the smaller outlet area (denoted O). We can apply the continuity equation (conservation of mass) and our knowledge of the nozzle inlet and outlet diameters to determine the flow velocity at the nozzle inlet.		$Q = A_O \cdot V = A_I \cdot V_I$ $V_I = \frac{A_O}{A_I} \cdot V = \left(\frac{d}{D}\right)^2 \cdot V$ $V_I = 3.39 \cdot \frac{m}{s}$
Energy	We can apply Bernoulli's equation (pages 40-41) to the flow through the smooth, horizontal nozzle. The knowledge of the inlet and outlet velocities allows us to determine the gage pressure at the nozzle inlet.		$P_I = \frac{\rho}{2} \cdot \left(V^2 - V_I^2\right)$ $P_I = 702 \cdot kPa$ $P_I = 101.8 \cdot psi$ $P_I = 235 \cdot ft\left(H_2O\right)$

Control volume, Pumps, Power	Now that we know the conditions at the nozzle inlet, it makes sense to consider a control volume consisting of the pipe that runs from the reservoir to the nozzle. As the flow density is constant, the volumetric flow rate is constant throughout the pipe. We can use the given pressure increase across the pump, the given pump efficiency, and the flow rate to determine the pump power (page 41).		$\dot{W} = \frac{Q \cdot \Delta P_P}{\eta} = 26.6 \cdot kW$ $\dot{W} = 35.7 \cdot hp$
Similitude, viscosity, energy	In addition to producing the nozzle inlet pressure, the pump must overcome any losses associated with the flow of the water through the pipe. One of these losses is that associated of the friction of the fluid (viscous forces) with the pipe wall. The frictional losses can be determined from the friction factor, the pipe geometry, and the flow velocity (page 41). The friction factor depends on the pipe roughness and the flow Reynolds number (page 41). The Reynolds number depends on the flow velocity, the pipe diameter, and the kinematic viscosity of the fluid. As the pipe diameter is uniform, the flow velocity (continuity equation) must be everywhere equal to the velocity at the nozzle inlet. The kinematic viscosity of the water is given in the table on page 44. The Moody diagram on page 45 provides the friction factor as a function of Reynolds number and relative roughness.		$\mathrm{Re} = \frac{V_I \cdot D}{\nu} = 380{,}000$ $\frac{e}{D} = \frac{0.06 \cdot mm}{10 \cdot cm} = 0.0006$ From the Moody diagram (page 45), we can readily determine that the flow is turbulent and that the friction factor is roughly: $f = 0.0185$ The associated pressure loss is given by the Darcy equation (page 41): $\Delta P_f = f \cdot \rho \cdot \frac{L}{D} \cdot \frac{V_I^2}{2}$ $\Delta P_f = 42.4 \cdot kPa$

Minor losses	Additionally the pump must overcome any losses associated with the entrance effects at the protruding pipe entrance (denoted e) where the flow leaves the reservoir and enters the pipe. The minor losses associated with this situation are given on page 41.		$\Delta P_e = C \cdot \rho \cdot \frac{V_I^2}{2}$ $C = 0.8$ $\Delta P_e = 4.58 \cdot kPa$
Pressure, height, energy	The pump, in conjunction with the static pressure, at the pipe inlet, must also "raise" the fluid the given height. Using the Bernoulli equation (page 41), the static pressure at the pipe inlet can be determined. Note that the flow velocity is uniform in the pipe, simplifying our equation.		$P_e = P_I - \Delta P_p + \Delta P_f + \Delta P_e$ $+\rho \cdot g \cdot H + \frac{1}{2} \cdot \rho \cdot V_I^2$ $H = 10 \cdot m$ $P_e = 52.0 \cdot kPa$
Pressure, height	The reservoir is large enough that the fluid contained therein can be treated as stationary. The height of fluid above the pipe inlet must be sufficient to generate the required pressure. The pressure is linearly dependent on height (page 39).		$P_e = \rho \cdot g \cdot H_W$ $H_W = \frac{P_e}{\rho \cdot g} = 5.32 \cdot m$
Forces on submerged surfaces, center of pressure, moment, two force member.	The gate on the left side of the reservoir is pivoted at a bottom point. The gate is prevented from opening by a strut that is attached to the gate at a point above the gate pivot. The forces exerted by the strut and the water on the gate must be such that the net moment about the gate pivot is zero. The strut is a two-force member so that any force exerted by the strut is parallel to its length. The moment of the strut force about the gate pivot is the horizontal component of the force multiplied by the vertical distance of the strut attachment point above the pivot. The net moment exerted by the water can be determined using the results from page 39.		Water force: $\frac{1}{2} \cdot \rho \cdot g \cdot \left(H_W + h\right) \cdot w \cdot \left(H_W + h\right)$ Location below water center: $\frac{\frac{1}{12} \cdot w \cdot \left(H_W + h\right)^3}{w \cdot \left(H_W + h\right) \cdot \frac{\left(H_W + h\right)}{2}}$ Strut force from moment: $S = \frac{1}{6} \cdot \frac{\rho \cdot g \cdot w \cdot \left(H_W + h\right)^3}{\cos\left(\theta\right) \cdot Y}$ $S = 634 \cdot kN$